Acoustic Propagation in Continental Shelf Break and Slope Environments

David Paul Knobles
Applied Research Laboratories
The University of Texas at Austin
PO Box 8029
Austin, TX 78713
phone: (512) 835-3687 fax: (512) 835-3259 email: knobles@arlut.utexas.edu

Award Number: N00014-10-1-0033
http://www.onr.navy.mil/sci_tech/32/321/ocean_acoustics.asp

LONG-TERM GOALS

The long-term goal of the research is to increase the physical understanding of acoustic propagation in continental shelf and slope environments in the 50-4000 Hz band. This includes both the physics of the seabed and the coupling to physical mechanisms in the water column in complex range- and azimuth-dependent littoral waveguides.

OBJECTIVES

There were two main objectives of the current research. The first objective was to develop and test a new coupled-mode approach that can serve as the basis for an advanced seabed attenuation analysis of broadband propagation data. The second objective was to apply the coupled mode approach to investigate the physics of scattering from a rough seabed surface.

APPROACH

A MATLAB numerical algorithm[1] was constructed for a two-way integral equation coupled mode approach that was originally programmed in FORTRAN.[2-3] The MATLAB algorithm includes a more efficient numerical method to compute the range-dependent mode coupling matrices.[4-5] Then, the MATLAB algorithm was modified to explicitly compute the forward and backward going coupled amplitudes, as discussed in Ref. 6. Next, the modal power flow within the trapped spectrum, into the continuum, and into the backward propagating modal spectrum was examined using the *master* equation from statistical mechanics, as discussed by Dozier and Tappert[7] and Colosi and Morozov.[8]

The propagation and scattering theory parallels Ref. 6. The 2-D coupled mode equations of dimension 2*N* for the forward and backward modal amplitudes ($R^{\infty}(r)=[R_1^{\infty}, R_2^{\infty}, \ldots, R_{N-1}^{\infty}, R_N^{\infty}]$ and $R^{0}(r)=[R_{N+1}^{0}, R_{N+2}^{0}, \ldots, R_{N+N-1}^{0}, R_{2N}^{0}]$) in cylindrical coordinates are

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **2012**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**

4. TITLE AND SUBTITLE **Acoustic Propagation in Continental Shelf Break and Slope Environments**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Applied Research Laboratories The University of Texas at Austin PO Box 8029 Austin, TX 78713**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S)
	11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**SAR**	**6**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

$$
\begin{aligned}
R_n^{\infty}(r) &= \Gamma_n^{\infty}(r) + \int_0^{r} r' G_n^{\infty}(r,r') \sum_{m=1}^{N} C_{nm}(r') R_m^{\infty}(r') dr' + \int_0^{r} r' G_n^{\infty}(r,r') \sum_{m=N+1}^{2N} C_{nm}(r') R_m^{0}(r') dr' \\
R_n^{0}(r) &= \Gamma_n^{0}(r) + \int_r^{\infty} r' G_n^{0}(r,r') \sum_{m=1}^{N} C_{nm}(r') R_m^{\infty}(r') dr' + \int_r^{\infty} r' G_n^{0}(r,r') \sum_{m=N+1}^{2N} C_{nm}(r') R_m^{0}(r') dr'
\end{aligned}, \tag{1}
$$

where

$$\Gamma_n^{\infty}(r) = \int_0^{r} r' G_n^{\infty}(r,r') \rho_n(r') dr'$$

$$\Gamma_n^{0}(r) = \int_r^{\infty} r' G_n^{0}(r,r') \rho_n(r') dr'.$$

ρ_n is the modal source function for the n*th* mode, and C_{nm} is the mode coupling matrix operator. *N* is the total number of modes, including both trapped and leaky modes. G_n^0 and G_n^{∞} are the Green's functions that satisfy

$$[\frac{\partial^2 r}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + k_n^2(r)] G_n(r,r') = -\frac{\delta(r-r')}{r}, \tag{2}$$

where $k_n(r)$ is the horizontal wavenumber eigenvalue for the n*th* mode at range *r*. When R^0 is neglected in Eq. 1, the result is a *one-way* equation for R^{∞}.

Having a solution to Eq. 1 permits the modal power flow to be calculated. Following Dozier and Tappert[7], one can, using the Born approximation and neglecting backscatter, write a coupled diffusion equation (called the *master equation)* for the modal amplitudes

$$\partial / \partial r \lhd |R_n(r)|^2 > = \sum_m C_{nm} [|R_m(r)|^2 - |R_n(r)|^2] - 2\alpha |R_n(r)|^2, \tag{3}$$

where an attenuation term has been added, as discussed by Colosi and Morozov.[8] We will show the results of constructing $\partial / \partial r \lhd |R_n(r)|^2 >$ (LHS of Eq. 3) and $\sum_m C_{nm} [|R_m(r)|^2 - |R_n(r)|^2] - 2\alpha |R_n(r)|^2$ (RHS of Eq. 3) from both the full two-way solution and the approximate one-way solution to Eq. 1. If the RHS is a good approximation to the LHS of Eq. 3, then one might expect to observe, in the limit of small attenuation, an equal partitioning of modes from the scattering.[7]

WORK COMPLETED

The work completed in FY12 included the application of the split integral equation coupled mode methodology to quantify the modal power flow for selected seabed roughness realizations. A potential important discovery was that for a seabed roughness characterized by a *relatively* small correlation length the energy lost from the trapped modes to the continuum and backscattering can be less than the energy *saved* by a transfer of energy within the trapped spectrum to the lower order modes with smaller

attenuation. The nature of this type of seabed scattering can cause the modal energy spectrum not to be evenly distributed as would be predicted by a *master* equation approach from statistical physics.

RESULTS

Selected numerical computations from the IECM approach are presented to illustrate scattering mechanisms resulting from a rough seabed. The background waveguide is 50 m deep with a stratified water sound speed profile (SSP) over an infinite half-space. The SSP has a thermocline depth at about 20 m. The geoacoustic parameter values describing the bottom half-space are 1645 m/s, 2.0 g/cm^3, and 0.4 dB/λ for the sediment compressional sound speed, the density, and the attenuation, respectively. The maximum (computational) range in the waveguide is 10 km, and the source is located at $r = 0$ m and depth 10 m. The roughness wavenumber power spectrum employed to generate a 1-D rough seabed realization is

$$P(k) = \frac{K_L h^2}{\pi (K_L^2 + k^2)^{\gamma/2}}, \tag{4}$$

where h, K_L, γ, and k are the rms roughness height, the roll-off wavenumber given by the reciprocal of the roughness correlation length, the scattering exponent, and the spatial wavenumber, respectively. For the current study scattering effects were computed at γ=2.0 and h = 0.5 m with K_L varying from 0.0025 1/m to 0.1 1/m. For the numerical computations the roughness power spectrum is employed to generate a rough seabed for 3.0 km < r < 7.0 km, and in the regions 0.0 km < r < 3.0 km and 7.0 km < r < 10.0 km the waveguide is range-invariant.

Figure 1 shows $|R_n(r)|$ (adjusted for cylindrical spreading) obtained from the full solution of the two-way coupled integral equations for an acoustic frequency of 200 Hz. For this waveguide and frequency there are 5 trapped modes and 8 continuum (leaky) modes for a total of 13 modes. The energy in the trapped modes is clearly shown to scatter into both the continuum modes for 3.0 km < r < 7.0 km and into the backward propagating modes (modes 14-26 on the vertical axis).

Figure 2 shows $\Delta(K_L)$ where Δ is defined as the 2-way depth-integrated transmission loss (DITL) minus the adiabatic (no mode coupling) DITL at 10 km. Δ becomes negative at about K_L = 0.06 1/m and continues to decrease for increasing K_L. An examination of the modal amplitudes showed that more energy was being scattered into mode 3 than into the continuum and into backscatter. The distance between interference peaks for modes 3 and 4 is $ID_{34} = 2\pi/|\mathrm{Re}(k_3) - \mathrm{Re}(k_4)| \approx 70$ m. For small K_L, ID_{34} is small compared to $1/K_L$. As K_L increases the spatial correlation length decreases and becomes on the order of ID_{34} and the interference length for other modal combinations within the trapped spectrum. More generally, when the modal interference distance is small compared to $1/K_L$ the large number of modal interference cycles over a correlation length allow for a significant transfer of energy between these modes to occur. Such scattering will be favored over pairs of modes that have a larger ID value. However, as K_L increases the correlation distance decreases and the preference for scattering into the continuum from the trapped spectrum diminishes. To establish the uncertainty in Fig. 2, the coupled mode equations were solved for 20 roughness realizations for selected K_L values. The result was that σ = 0.76 at K_L = 0.1 1/m and σ = 0.48 at K_L =0.018 1/m.

Figure 3a shows $\partial / \partial r \langle |R_n(r)|^2 \rangle$, where the modal amplitudes were those used to construct Fig. 1. Figure 3b shows $\partial / \partial r \langle |R_n(r)|^2 \rangle$ computed with the RHS of Eq. 3. A comparison of Figs. 3a and 3b suggests that the RHS of Eq. 3 overestimates the amount of acoustic intensity radiated into the continuum; however, the RHS of Eq. 3 underestimates the amount of energy transferred into backscatter. Some of the differences between Figs. 3a and 3b may be ascribed to the inclusion of backward propagation into the RHS of Eq. 3, which is neglected in the original derivation of Eq. 3 by Dozier and Tappert. Figure 3b suggests an equal partitioning of modes (trapped and continuum) whereas Fig. 3a clearly suggests that equal partitioning is not occurring.

Figure 4 is the same as Fig. 3, except the one-way approximation to Eq. 1 was used to compute the modal amplitudes. The one-way approximation is more consistent with the approximations used to derive Eq. 3, namely, the neglect of the backscattered acoustic field. Although one observes qualitative agreement of Figs. 4a and 4b, equal partitioning of modes in Fig. 4a occurs to a lesser extent than in Fig. 4b. One might make the tentative conclusion that the RHS of Eq. 3, based on a Born-like approximation, does not adequately capture the physics of scattering for seabed roughness having a small correlation length as compared to modal interference lengths in the trapped spectrum. The relatively smaller correlation lengths can inhibit energy from being transferred from the trapped to the continuum, which in turn inhibits equal partitioning of modes.

The above are only tentative conclusions and require additional study of how to properly apply the *master* equation for waveguides that exhibit strong variability.

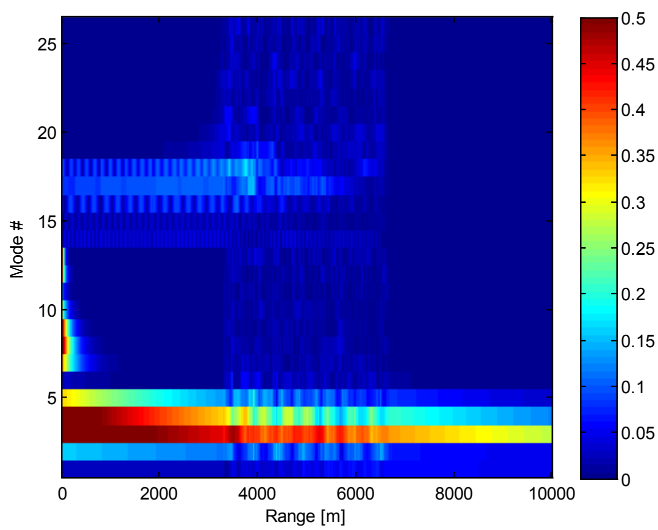

FIGURE 1. Modal amplitudes versus range.

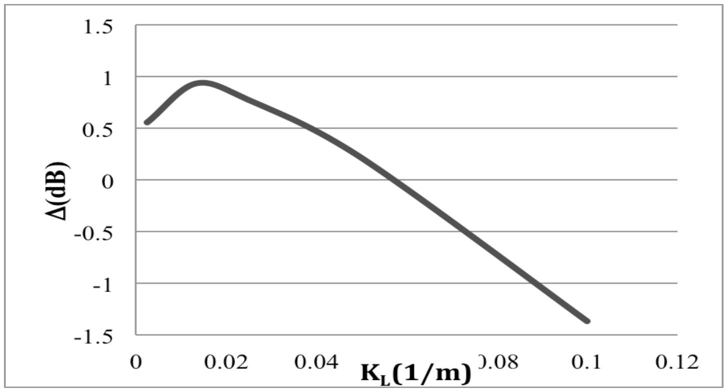

FIGURE 2. $\Delta(K_L)$ for h= 0.5 m.

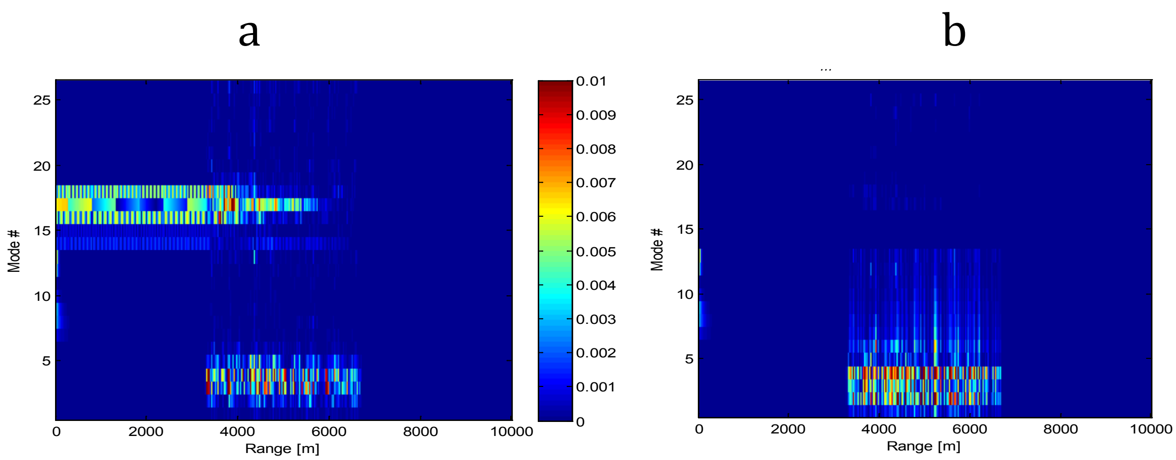

FIGURE 3. Modal power flow predicted by two-way solution in (a) LHS of Eq. 3 and (b) RHS of Eq. 3.

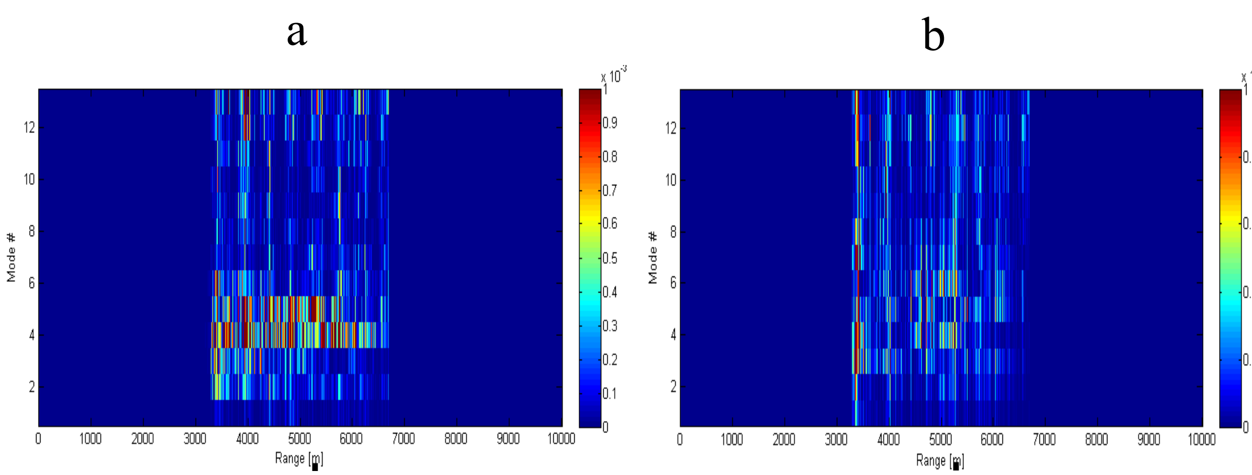

FIGURE 4. Modal power flow predicted by one-way solution in (a) LHS of Eq. 3 and (b) RHS of Eq. 3.

IMPACT/APPLICATIONS

One potential impact of this research is that these studies may assist in understanding how to optimally combine advanced propagation models (non-separable and 3-D) and information inference methods as one proceeds to study ocean waveguides with increasing complexity and inhomogeneity.

TRANSITIONS

The combined study of statistical inference and the effects of seabed scattering is expected to relate propagation statistics to physical mechanisms on continental shelf and slope environments. The knowledge of such statistics and their relationship to the physics of the propagation may be useful for sonar applications in continental shelf and slope environments.

RELATED PROJECTS

Related research projects include applying maximum entropy to the statistical inference of values for seabed geoacoustic and scattering parameters from measured data.

REFERENCES

1. J. D. Sagers, *Integral Equation Coupled Mode Code* (Version 2.0) [MATLAB program]. ARL:UT internal software (Accessed August 2012).
2. D. P. Knobles, "Solutions of coupled-mode equations with a large dimension in underwater acoustics," J. Acoust. Soc. Am. **96**, 1741 (1994).
3. D. P. Knobles, S. A. Stotts, R. A. Koch, and T. Udagawa, "Integral equation coupled mode approach applied to internal wave problems," J. Comp. Acoust. **9**, 149-167 (2001).
4. R. A. Koch, "Coupled mode formalism in three dimensions," ARL-TL-EV-10-01 (2010).
5. Jason D. Sagers, "Predicting Acoustic Intensity Fluctuations Induced by Nonlinear Internal Waves in a Shallow Water Waveguide," Ph.D. Thesis, University of Texas at Austin, 2012.
6. D. P. Knobles and J. D. Sagers, "A nonlocal effective operator for coupling forward and backward propagating modes in inhomogeneous media," J. Acoust. Soc. Am. **130** 2673-2680 (2011).
7. L. B. Dozier and F. D. Tappert, "Statistics of normal mode amplitudes in a random ocean. I. Theory," J. Acoust. Soc. Am. **63** 353-365 (1978).
8. J. A. Colosi and A. K. Morozov, "Statistics of normal mode amplitudes in an ocean with random sound speed perturbations: Cross-mode coherence and mean intensity," J. Acoust. Soc. Am. **126** 1026-1035 (2009).

PUBLICATIONS

1. D. P. Knobles and J. D. Sagers, "A nonlocal effective operator for coupling forward and backward propagating modes in inhomogeneous media," J. Acoust. Soc. Am. **130** 2673-2680 (2011).
2. D. P. Knobles, J. D. Sagers, and R. A. Koch, "Maximum entropy approach for statistical inference in an ocean acoustic waveguide," J. Acoust. Soc. Am **131** 1087-1101 (2012).